Scherk-Schwarz SUSY breaking from the viewpoint of 5D conformal supergravity

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Abstract

We reinterpret the Scherk-Schwarz (SS) boundary condition for $SU(2)_R$ in a compactified five-dimensional (5D) Poincaré supergravity in terms of the twisted $SU(2)_U$ gauge fixing in 5D conformal supergravity. In such translation, only the compensator hypermultiplet is relevant to the SS twist, and various properties of the SS mechanism can be easily understood. Especially, we show the correspondence between the SS twist and constant superpotentials within our framework.

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1   Introduction

Supersymmetry (SUSY) is considered as a fundamental symmetry in superstring theory. It is also one of the most promising candidates for the physics beyond the standard model. For example, it protects the weak scale against large radiative corrections. However the low-energy particle contents and interactions do not respect SUSY, and thus it must be broken above the weak scale.

The Scherk-Schwarz (SS) mechanism [1] of SUSY breaking has been revisited recently as a phenomenologically interesting candidate for the physics beyond the standard model [2]. The most simple setup in such context was constructed within the framework of five-dimensional (5D) supergravity compactified on an orbifold S^1/Z_2. It was argued that the SS mechanism can be regarded as a spontaneous breaking of SUSY [2, 3, 4]. This argument was based on the fact that the SS twist corresponds to a nonvanishing Wilson line [3] for the auxiliary $SU(2)_{\boldsymbol{U}}$ gauge field in the off-shell supergravity. This nonvanishing Wilson line effect appears as the auxiliary component of the so-called radion superfield in the $N = 1$ superspace description. A lot of analyses based on this description were done in literatures [2, 3, 4]. It was also discussed that boundary mass terms yield the same spectrum as the SS breaking, and thus can be interpreted as an equivalent description to the SS twist [5]. On the other hand, it has been argued that the SS twist leads to an inconsistent theory on the warped geometry [6] if SUSY is a local symmetry.

In this paper we reinterpret the Scherk-Schwarz (SS) boundary condition for $SU(2)_R$ in the compactified five-dimensional (5D) Poincaré supergravity as the twisted $SU(2)_{\boldsymbol{U}}$ gauge fixing in 5D conformal supergravity. In such an interpretation, only the compensator hypermultiplet is relevant to the SS breaking. Starting from a hypermultiplet compensator formalism of the 5D conformal supergravity [7, 8, 9, 10], we construct the 5D Poincaré supergravity with SS twist. By this construction, various properties of the SS mechanism can be easily understood. Here, we will explicitly show the Wilson line interpretation of the SS twist [3], the correspondence between the twist and constant superpotentials [5], and the inconsistency of the twist in the warped background [6] within our framework. We will also derive the $N = 1$ superspace description of the SS twist based on works [11, 12, 13, 14], in which the $N = 1$ superfields are directly derived from the 5D superconformal multiplets.

In Sec. 2, we first review the hypermultiplet compensator formulation of 5D conformal supergravity [7, 8, 9, 10]. In Sec. 3, we introduce the twisted $SU(2)_{\boldsymbol{U}}$ gauge fixing and derive the 5D Poincaré supergravity with SS twist. Here we show the Wilson line interpretation and the correspondence between the SS twist and the boundary constant superpotentials. We also demonstrate what happens if we consider the twist on the warped geometry. In Sec. 4, we derive the $N = 1$ superspace description of the SS twist within our framework. Finally we summarize our results and give some discussions in Sec. 5. Some detailed expressions for the 5D conformal supergravity in our notation are exhibited in Appendix A based on Refs. [7, 8, 9, 10].

2 Review of hypermultiplet compensator formalism

In this section we briefly review the hypermultiplet compensator formulation of 5D conformal supergravity derived in Refs. [7, 8, 9, 10]. The 5D superconformal algebra consists of the Poincaré symmetry $\boldsymbol{P}$, $\boldsymbol{M}$, the dilatation symmetry $\boldsymbol{D}$, the $SU(2)$ symmetry $\boldsymbol{U}$, the special conformal boosts $\boldsymbol{K}$, $N = 2$ supersymmetry $\boldsymbol{Q}$, and the conformal supersymmetry $\boldsymbol{S}$. We use $\mu, \nu, \ldots$ as five-dimensional curved indices and $m, n, \ldots$ as the tangent flat indices. The gauge fields corresponding to these generators $\boldsymbol{X}_A = \boldsymbol{P}_m, \boldsymbol{M}_{mn}, \boldsymbol{D}, \boldsymbol{U}_{ij}, \boldsymbol{K}_m, \boldsymbol{Q}_i, \boldsymbol{S}_i$, are respectively $h_\mu^A = e_\mu^m, \omega_\mu^{mn}, b_\mu, V_\mu^{ij}, f_\mu^m, \psi_\mu^i, \phi_\mu^i$, in the notation of Refs. [7, 8, 9, 10]. The index $i = 1, 2$ is the $SU(2)_{\boldsymbol{U}}$-doublet index which is raised and lowered by antisymmetric tensors $\epsilon^{ij} = \epsilon_{ij}$.

In this paper we are interested in the following superconformal multiplets:

- 5D Weyl multiplet: $(e_\mu^m, \psi_\mu^i, V_\mu^{ij}, b_\mu, v^{mn}, \chi^i, D)$,

- 5D vector multiplet: $(M, W_\mu, \Omega^i, Y^{ij})^I$,

- 5D hypermultiplet: $(\mathcal{A}^\alpha_i, \zeta^\alpha, \mathcal{F}^\alpha_i)$.

Here the index $I = 0, 1, 2, \ldots, n_V$ labels the vector multiplets, and $I = 0$ component corresponds to the central charge vector multiplet[1]. For hyperscalars $\mathcal{A}^\alpha_i$, the index α runs as $\alpha = 1, 2, \ldots, 2(p + q)$ where p and q are the numbers of the quaternionic compensator and the physical hypermultiplets respectively. In this paper we adopt the single compensator case, $p = 1$, and separate the indices such as $\alpha = (a, \underline{\alpha})$ where $a = 1, 2$ and $\underline{\alpha} = 1, 2, \ldots, 2q$ are indices for the compensator and the physical hypermultiplets respectively.

The superconformal gauge fixing for the reduction to 5D Poincaré supergravity is given by

$$
\begin{aligned}
\boldsymbol{D} &: \quad \mathcal{N} = M_5^3 \equiv 1, \\
\boldsymbol{U} &: \quad \mathcal{A}^a_i \propto \delta^a_i, \qquad (p = 1) \\
\boldsymbol{S} &: \quad \mathcal{N}_I \Omega^{Ii} = 0, \\
\boldsymbol{K} &: \quad \mathcal{N}^{-1} \hat{\mathcal{D}}_m \mathcal{N} = 0,
\end{aligned}
\tag{1}
$$

where $\mathcal{N} = C_{IJK} M^I M^J M^K$ is the norm function of 5D supergravity with a totally symmetric constant C_{IJK}, and $\mathcal{N}_I = \partial \mathcal{N} / \partial M^I$. The derivative $\hat{\mathcal{D}}_m$ denotes the superconformal covariant derivative. Here and hereafter we take the unit that the 5D Planck scale is unity, $M_5 = 1$.

The invariant action for 5D supergravity on S^1/Z_2 is given as [9]

$$
S = \int d^4x \int dy \, (\mathcal{L}_b + \mathcal{L}_f + \mathcal{L}_{\text{aux}} + \mathcal{L}_{N=1}),
$$

where $\mathcal{L}_b$, $\mathcal{L}_f$, $\mathcal{L}_{\text{aux}}$ and $\mathcal{L}_{N=1}$ are the Lagrangians for the bosonic, fermionic, auxiliary fields and the boundary Lagrangian respectively, which are shown in Eqs. (24) and (25) in Appendix A. It was shown in Refs. [9, 10] that the above off-shell supergravity can be consistently compactified on an orbifold S^1/Z_2 by the Z_2-parity assignment shown in Table 1 in Appendix A.

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[1]Roughly speaking, the vector field in this multiplet corresponds to the graviphoton.

3 $SU(2)_U$ gauge fixing and Scherk-Schwarz twist

In this section, starting from the hypermultiplet compensator formalism reviewed in the previous section, we construct the 5D Poincaré supergravity with SS twist by introducing the twisted $SU(2)_U$ gauge fixing. In the following subsections, we will explicitly show the Wilson line interpretation, the correspondence between the twist and the boundary constant superpotentials and the inconsistency of the twist on the warped geometry within our framework.

First we recall the usual SS mechanism. The following arguments are based on the 5D Poincaré supergravity on the orbifold S^1/Z_2 defined by

$$\Phi(x, -y) = \pm Z\Phi(x, y),$$

where Z acts on the $SU(2)_R$ indices of fields and is chosen as $Z = \sigma_3$ without a loss of generality. The SS twist is defined as

$$\Phi(x, y + 2\pi R) = T\Phi(x, y),$$

where R is the radius of the fifth dimension y and T also acts on the $SU(2)_R$ indices that is given by

$$T = e^{-2\pi i \vec{\omega} \cdot \vec{\sigma}}.$$

The twist vector $\vec{\omega} = (\omega_1, \omega_2, \omega_3)$ determines the strength and the direction of the SS twist and $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. For $Z = \sigma_3$, the consistency condition

$$TZT = Z, \tag{2}$$

requires $\omega_3 = 0$ (except for some special cases [3]). We can go to the periodic field basis by redefining fields [3, 4] as $\Phi \to e^{i\vec{\omega} \cdot \vec{\sigma} f(y)} \Phi$ where $e^{i\vec{\omega} \cdot \vec{\sigma} f(y)}$ again acts on the $SU(2)_R$ indices and $f(y)$ satisfies

$$f(y + 2\pi R) = f(y) + 2\pi. \tag{3}$$

3.1   Twisted $SU(2)_U$ gauge fixing

In the derivation of 5D Poincaré supergravity from the 5D conformal supergravity using the hypermultiplet compensator, the $SU(2)_R$ symmetry is defined as the diagonal subgroup of the direct product between the original $SU(2)_U$ gauge symmetry and $SU(2)_C$ which rotates the compensator index a,

$$SU(2)_U \times SU(2)_C \to SU(2)_R, \tag{4}$$

through the U-gauge fixing $\mathcal{A}^a{}_i \propto \delta^a_i$. The D- and U-gauge fixings in Eq. (1) completely fix the quaternionic compensator hyperscalar field as[2]

$$\mathcal{A}^a{}_i \equiv \delta^a_i \sqrt{1 + \mathcal{A}^{\underline{\alpha}}{}_i \mathcal{A}_{\underline{\alpha}}{}^i}. \tag{5}$$

---

[2]Note that the equations of motion for the auxiliary fields D' and χ' in Eq. (24) in Appendix A result in $\mathcal{A}^2 = 2\mathcal{N}$ and $\mathcal{A}^{\underline{\alpha}}{}_i \zeta_\alpha \equiv \mathcal{A}^a{}_i \zeta_a - \mathcal{A}^{\underline{\alpha}}{}_i \zeta_{\underline{\alpha}} = 0$ respectively.

Then the compensator fermion ζ^a is fixed by

$$\zeta_a \delta^a{}_i = \zeta_{\underline{\alpha}} \frac{\mathcal{A}^{\underline{\alpha}}{}_i}{\sqrt{1 + \mathcal{A}^{\underline{\beta}}{}_j \mathcal{A}_{\underline{\beta}}{}^j}}. \tag{6}$$

However, if we consider a torus compactification of the fifth dimension y with the radius R, we have inequivalent classes of the $SU(2)_{\boldsymbol{U}}$ gauge fixing which is parameterized by a twist vector $\vec{\omega}$ as

$$\mathcal{A}^a{}_i \equiv \left(e^{i\vec{\omega}\cdot\vec{\sigma} f(y)} \right)^a{}_i \sqrt{1 + \mathcal{A}^{\underline{\alpha}}{}_i \mathcal{A}_{\underline{\alpha}}{}^i}, \tag{7}$$

and then

$$\zeta_a \left(e^{i\vec{\omega}\cdot\vec{\sigma} f(y)} \right)^a{}_i = \zeta_{\underline{\alpha}} \frac{\mathcal{A}^{\underline{\alpha}}{}_i}{\sqrt{1 + \mathcal{A}^{\underline{\beta}}{}_j \mathcal{A}_{\underline{\beta}}{}^j}}, \tag{8}$$

where $f(y)$ is a function satisfying Eq. (3). This twist vector $\vec{\omega}$ specifies the class of torus compactified 5D Poincaré supergravity and $\omega_{1,2} \neq 0$ corresponds to the SS twist parameter. In other words, from Eq. (4), the SS boundary condition for all the fields with $SU(2)_R$ index in the Poincaré supergravity is simply (equivalently) given by only the compensator $(SU(2)_C)$ twisting in the framework of conformal supergravity [12]. We usually choose $f(y) = y/R$, but here we do not specify its explicit form for the later convenience. Note that the different choice of $f(y)$ with the same $\vec{\omega}$ gives the same physics, because it corresponds to the $\boldsymbol{U}$-gauge fixing parameter.

In the superconformal formulation, $SU(2)_{\boldsymbol{U}}$ gauge field on-shell (without the boundary action) is given by

$$V^{ij}_{\mu\,\mathrm{sol}} = -\frac{1}{2\mathcal{N}} \left(2\mathcal{A}^{\bar{\alpha}(i}\nabla_\mu \mathcal{A}_\alpha{}^{j)} + i\mathcal{N}_{IJ}\bar{\Omega}^{Ii}\gamma_\mu \Omega^{Jj} \right), \tag{9}$$

where $\mathcal{N}_{IJ} = \partial\mathcal{N}/\partial M^I \partial M^J$. The notations (i, j) and $\bar{\alpha}$ in the superscript are defined respectively in Eqs. (27) and (28) in Appendix A. Under the gauge fixing (7), we go to the periodic field basis by

$$\mathcal{A}^a{}_i \rightarrow U^a{}_b(y)\mathcal{A}^b{}_i, \qquad \zeta^a \rightarrow U^a{}_b(y)\zeta^b, \tag{10}$$

where

$$U^a{}_b(y) \equiv \left(e^{-i\vec{\omega}\cdot\vec{\sigma} f(y)} \right)^a{}_b.$$

In this basis all the field (including the compensator) has a usual periodic boundary condition. The $SU(2)_{\boldsymbol{U}}$ gauge fixing (7) reduces into trivial one (5), while Eq. (9) becomes

$$V^{ij}_{y\,\mathrm{sol}} \rightarrow -\frac{1}{2\mathcal{N}} \left(2\mathcal{A}^{\bar{\alpha}(i}\nabla_y \mathcal{A}_\alpha{}^{j)} + ia_{IJ}\bar{\Omega}^{Ii}\gamma_y \Omega^{Jj} \right) + \frac{1}{\mathcal{N}}(U_{ca}\partial_y U^c{}_b)\mathcal{A}^{a(i}\mathcal{A}^{bj)}. \tag{11}$$

Namely we find that the twisted $SU(2)_{\boldsymbol{U}}$ gauge fixing (7) yields a field-independent shift of $SU(2)_{\boldsymbol{U}}$ gauge field V^{ij}_y. It leads to a nonvanishing contribution to the Wilson line,

$$\int_0^{2\pi R} dy\, V_y{}^i{}_j = 2\pi i(\vec{\omega}\cdot\vec{\sigma})^i{}_j + \cdots,$$

where we have adopted the property (3) and the ellipsis denotes the contributions from the physical fields. Then we find that the SS SUSY breaking can be interpreted as the breaking caused by a nonvanishing Wilson line of $SU(2)_U$ gauge field. As shown in Ref. [3] (based on the linear multiplet compensator formalism [15]), V_y^{ij} appears in the auxiliary component of the so-called radion superfield in the $N = 1$ superspace description. We will show this within our framework in Sec. 4. (See Eq. (23).)

Next we derive SUSY breaking terms induced by the SS twist in the periodic basis. By the field redefinition (10), the compensator fixing conditions become normal ones (5) and (6) respectively, while we have additional $\vec{\omega}$ dependent terms which arise through the y-derivatives of the compensator fields in the action shown in Eq. (24) in Appendix A. They are given by

$$
\begin{aligned}
e^{-1}\mathcal{L}_\omega &= f'(y)(i\vec{\omega}\cdot\vec{\sigma})_{ab}\Big\{\epsilon^{ij}(\mathcal{A}^a{}_j\nabla_4\mathcal{A}^b{}_i - \mathcal{A}^b{}_i\nabla_4\mathcal{A}^a{}_j) + 2i\bar\zeta^b\gamma^4\zeta^a \\
&\quad -4i\bar\psi_m^i\gamma^4\gamma^m\zeta^a\mathcal{A}^b{}_i + 2i\bar\psi_m^{(i}\gamma^{m4n}\psi_n^{j)}\mathcal{A}^b{}_j\mathcal{A}^a{}_i - \frac{1}{\mathcal{N}}\epsilon^{jk}V_4{}^i{}_k(\mathcal{A}^b{}_i\mathcal{A}^a{}_j + \mathcal{A}^b{}_j\mathcal{A}^a{}_i)\Big\} \\
&\quad -(f'(y)|\vec{\omega}|)^2\epsilon^{ij}\epsilon_{ab}\mathcal{A}^b{}_i\mathcal{A}^a{}_j.
\end{aligned}
$$

where $\epsilon^{ij} = \epsilon_{ij} = i\sigma_2$. We remark that the $\vec{\omega}$ dependent gaugino mass will be generated after integrating out the auxiliary field V_4.

The V_4 is included in the auxiliary Lagrangian in Eq. (24) as

$$
e^{-1}\mathcal{L}^V_{\text{aux}} = -\eta^{mn}(V_m - V_{m\,\text{sol}})^{ij}(V_n - V_{n\,\text{sol}})_{ij},
$$

where $V_{m\,\text{sol}} = e_m^\mu V_{\mu\,\text{sol}}$ is given by Eq. (9). Then $\mathcal{L}_\omega$ on-shell becomes

$$
\begin{aligned}
e^{-1}\mathcal{L}_\omega^{\text{on-shell}} &= f'(y)(i\vec{\omega}\cdot\vec{\sigma})_{ij}\Big\{2i\bar\zeta^{\underline\beta}\gamma^4\zeta^{\underline\alpha}\mathcal{A}_{\underline\beta}{}^j\mathcal{A}_{\underline\alpha}{}^i(1 + \mathcal{A}_{\underline\alpha}{}^k\mathcal{A}^{\underline\alpha}_k)^{-1} + 4i\bar\psi_m^i\gamma^4\gamma^m\zeta^{\underline\alpha}\mathcal{A}_{\underline\alpha}{}^j \\
&\quad + \Big(2i\bar\psi_m^{(i}\gamma^{m4n}\psi_n^{j)} - 2\mathcal{A}^{\underline\alpha(i}\nabla_4\mathcal{A}_{\underline\alpha}{}^{j)} + ia_{IJ}\bar\Omega^{Ii}\gamma_4\Omega^{Jj}\Big)(1 + \mathcal{A}_{\underline\alpha}{}^k\mathcal{A}^{\underline\alpha}_k)\Big\} \\
&\quad -2(f'(y)|\vec{\omega}|)^2(\mathcal{A}_{\underline\alpha}{}^i\mathcal{A}^{\underline\alpha}_i + (\mathcal{A}_{\underline\alpha}{}^i\mathcal{A}^{\underline\alpha}_i)^2),
\end{aligned}
$$

where we have applied the compensator fixings (5) and (6). Then the important part for the low energy physics up to quadratic in fields is given by

$$
e^{-1}\mathcal{L}_\omega^{\text{mass}} = f'(y)(i\vec{\omega}\cdot\vec{\sigma})_{ij}\Big(2i\bar\psi_m^{(i}\gamma^{m4n}\psi_n^{j)} + ia_{IJ}\bar\Omega^{Ii}\gamma_4\Omega^{Jj}\Big) - 2(f'(y)|\vec{\omega}|)^2\mathcal{A}_{\underline\alpha}{}^i\mathcal{A}^{\underline\alpha}_i,
$$

which contains the mass terms of the gravitino, the gauginos and the hyperscalars [2, 3, 4]. We remark that the cosmological constant proportional to $|\vec{\omega}|^2$ in $\mathcal{L}_\omega$ has been cancelled on-shell by the equation of motion for V_4.

3.2 Singular gauge fixing and boundary interpretation

As we mentioned above, an explicit function form of $f(y)$ does not affect the physical consequence because $f(y)$ is just a gauge fixing parameter. We usually choose $f(y) = y/R$ which gives the simplest description. However in this section, motivated by the argument of *generalized symmetry breaking* in Ref. [5], we choose a gauge fixing parameter,

$$
f(y) = \frac{\pi}{2}\sum_n (\text{sgn}(y - n\pi R) - \text{sgn}(-n\pi R)), \tag{12}
$$

5

where $\text{sgn}(y)$ is the sign-function. Namely,

$$f'(y) = \pi \sum_n \delta(y - n\pi R).$$

For this $f(y)$, $\mathcal{L}_\omega$ becomes totally boundary terms. However we should be careful about the singular structure of this gauge fixing parameter which can change physics depending on the regularization. In the following we concentrate on the contribution from the Z_2-even fields which does not suffer from this singularity. We will briefly mention about the correction from the Z_2-odd fields at the end of this section.

In the situation that the compensator is not charged under the physical gauge field (i.e. R-symmetry is not gauged), we introduce $N = 1$ invariant constant superpotentials W at the orbifold fixed points (see Eq. (25) in Appendix A)

$$\begin{aligned} \mathcal{L}_{N=1} &= \sum_n \delta(y - n\pi R) \left[\Sigma^3 W \right]_F \\ &= \sum_n \delta(y - n\pi R) e_{(4)} W \\ &\quad \times \left[2i\mathcal{A}^{a=2}_{i=2}(1 + iW^{I=0}_4/M^{I=0}) \tilde{\mathcal{F}}^{a=2}_{i=1} + 2\mathcal{A}^{a=2}_{i=2}(\nabla_4\mathcal{A} + \mathcal{A}V_4 - 2i\bar{\psi}_4\zeta)^{a=2}_{i=1} \right. \\ &\quad \left. - 2\bar{\zeta}_+ \mathcal{P}_R \zeta^{\hat\alpha=1}_+ + 4i\mathcal{A}^{a=2}_{i=2}\bar{\psi}_+ \cdot \gamma \mathcal{P}_R \zeta^{\hat\alpha=1}_+ - 2\mathcal{A}^{a=2}_{i=2}\mathcal{A}^{a=2}_{i=2}\bar{\psi}_{m+}\gamma^{mn}\mathcal{P}_L\psi_{n+} \right] + \text{h.c.}, \end{aligned}$$

where $\mathcal{P}_{R,L} \equiv (1 \pm \gamma_5)/2$, $\tilde{\mathcal{F}}^\alpha_i \equiv \mathcal{F}^\alpha_i - \mathcal{F}^\alpha_{i\,\text{sol}}$ and $\mathcal{F}^\alpha_{i\,\text{sol}}$ is shown in Eq. (26) in Appendix A. The Z_2-even fields ψ_{m+} and $\zeta^{\hat\alpha}_+$ are defined in Table 1 in Appendix A. The symbol $[\cdots]_F$ represents the F-term invariant formula in $N = 1$ superconformal tensor calculus [9] and $e_{(4)}$ is the determinant of the 4D induced vierbein. Here let us see the relation between the SS twist and the boundary constant superpotentials. Interestingly enough, we can show that, provided W satisfies

$$W = \pi(\omega_2 + i\omega_1), \tag{13}$$

the constant superpotentials generate exactly the same terms as (the Z_2-even part of) the previous SS twist Lagrangian $\mathcal{L}_\omega$ after integrating out compensator auxiliary field $\mathcal{F}^a_i$, that is,

$$\mathcal{L}_W + \mathcal{L}_{\text{aux}} = \mathcal{L}_\omega + \mathcal{L}_{\text{aux}}\big|_{\mathcal{F}^a_i \to \mathcal{F}^a_i + C^a_i}, \tag{14}$$

where

$$C^a_i = \pi \sum_n \delta(y - n\pi R) e^{-1} e_{(4)} \left(\frac{(M^{I=0})^{-1}W^{I=0}_4 \mathbf{1}_2 + \sigma_3}{1 + (M^{I=0})^{-2}(W^{I=0}_4)^2} \right)^a_{\ b} (i\vec\omega \cdot \vec\sigma)^b_{\ c} \mathcal{A}^c_i.$$

The Lagrangian $\mathcal{L}_{\text{aux}}$ is shown in Eq. (24) in Appendix A which consists of complete squares of the auxiliary fields including $\mathcal{F}^a_i$ and V^{ij}_y.

Therefore we conclude that, with the singular gauge fixing parameter (12), the Z_2-even part of the SS twist $\vec\omega = (\omega_1, \omega_2, 0)$ is equivalent to the $N = 1$ invariant constant superpotentials $W = \pi(\omega_2 + i\omega_1)$ at boundaries. From this correspondence between SS

twist and the constant superpotentials, we confirm that SUSY breaking caused by the SS twist is not explicit (at least for the Z_2-even part), because the boundary constant superpotential is $N = 1$ invariant. We remark that this correspondence has been easily found at the full supergravity level, not in the effective theory, thanks to our simplified interpretation of SS twist.

We should comment that with a suitable regularization of the δ-function, the parity odd part on the boundary modifies effectively the relation (13) as, e.g. $W = \tan(\pi\omega_2)$ for the σ_2 twist $\vec{\omega} = (0, \omega_2, 0)$, that was shown in Ref. [3].

3.3   Scherk-Schwarz twist and AdS$_5$ geometry

It was suggested in Ref. [6] that the SS twist yields an inconsistency in the supergravity on AdS$_5$ geometry. Therefore, in this section, we consider the SS twist in the AdS$_5$ background within our framework of twisted $SU(2)_U$ fixing, and see what happens. It is known that the gauging of $U(1)_R$ symmetry by the graviphoton is necessary to realize the AdS$_5$ geometry keeping SUSY [16, 17, 18]. In fact, the negative cosmological constant is proportional to the $U(1)_R$ gauge coupling in such a case. When a physical vector field $W_{R\mu}$ gauges the $U(1)_C$ subgroup of $SU(2)_C$,

$$\mathcal{D}_\mu \mathcal{A}^a{}_i \;\; = \;\; \partial_\mu \mathcal{A}^a{}_i - \mathcal{A}^{aj} V_{\mu ij} - (g_R W_\mu^R)^a{}_b \mathcal{A}^b{}_i + \cdots, \tag{15}$$

where

$$(g_R W_\mu^R)^a{}_b \;\; \equiv \;\; g_R W_\mu^R (\vec{q} \cdot i\vec{\sigma})^a{}_b \;\; = \;\; \begin{cases} |\vec{q}| g_R W_\mu^R (i\sigma_1 \sin\theta_R + i\sigma_2 \cos\theta_R)^a{}_b & (g_R : Z_2\text{-even}) \\ |\vec{q}| g_R W_\mu^R (i\sigma_3)^a{}_b & (g_R : Z_2\text{-odd}) \end{cases} ,$$

we shift the auxiliary $SU(2)_U$ gauge field by $V_{\mu ij}^N \equiv V_{\mu ij} + g_R W_{\mu ij}^R$ [7]. Then the covariant derivative (15) becomes

$$\mathcal{D}_\mu \mathcal{A}^a{}_i \;\; = \;\; \partial_\mu \mathcal{A}^a{}_i + \mathcal{A}^{aj} V_{\mu ij}^N - g_R \mathcal{A}^{aj} W_{\mu ij}^R - (g_R W_\mu^R)^a{}_b \mathcal{A}^b{}_i + \cdots. \tag{16}$$

The third and fourth term in the right-hand side of Eq. (16) cancels each other under the normal U-gauge fixing (5), and consequently this W_μ^R becomes the massless $U(1)_R$ gauge field.

However, with the nonvanishing SS twist $\vec{\omega}$ in the twisted U-gauge fixing (7), we find that the compensator decouples from this W_μ^R vector field in (16), when and only when the condition

$$[(\vec{q} \cdot i\vec{\sigma}), (\vec{\omega} \cdot i\vec{\sigma})] \;\; = \;\; 0, \tag{17}$$

is satisfied. Otherwise the $U(1)_R$ gauge field acquires a mass from the beginning (from the superconformal-covariant kinetic term of the compensator), which breaks the unitarity of this Poincaré supergravity. This means that the twist vector $\vec{\omega}$ should satisfy (17) in order to consistently gauge the $U(1)_R$ subgroup of $SU(2)_R$ symmetry defined by $\vec{q}$.

To obtain the GP-FLP [17] (BKVP [18]) model for a supersymmetric warped brane world, we need to gauge the $U(1)_R$ symmetry by the graviphoton with Z_2-odd gauge

coupling[3] g_R, i.e. $\vec{q} = |\vec{q}|(0,0,1)$. From the above argument, the only possible twist vector in this case is $\vec{\omega} = 0$, which does not cause SUSY breaking[4].

On the other hand in the ABN model [16] in which the $U(1)_R$ symmetry is gauged by the graviphoton with Z_2-even gauge coupling g_R, i.e. $\vec{q} = |\vec{q}|(\sin\theta_R, \cos\theta_R, 0)$, the possible twist vector is $\vec{\omega} = |\vec{\omega}|(\sin\theta_R, \cos\theta_R, 0)$. In this case we find a possibility to have the SS SUSY breaking in a slice of AdS_5. However it was pointed out in Refs. [10, 15] that ABN model is not derived from the known off-shell formulations with the linear multiplet [15] or the hypermultiplet compensator [10]. We can not find the Killing spinor on this background in those off-shell formulations for ABN model even without the SS twisting. This is still an open question.

4 $N = 1$ superspace description

In this section, we describe SS SUSY breaking in the $N = 1$ superspace formalism, which is directly derived from the 5D conformal supergravity [11, 12]. As we found in the previous section, the SS twist yields inconsistent theory when the R-symmetry is gauged by some physical gauge field. Then, in this section, we restrict ourselves to the case that the R-symmetry is not gauged, which means that the background geometry is flat (neglecting the backreaction from the VEVs of the physical fields). We first review the action shown in Ref. [14], and extend it to the case with twisted $SU(2)_U$ gauge fixing (7).

For simplicity, we consider the case that $n_V = 1$, $(p, q) = (1, 1)$ and the maximally symmetric norm function, $\mathcal{N} = (M^{I=0})^3 - M^{I=0}(M^{I=1})^2/2$. Then, the superspace action is written by the Lagrangian $\mathcal{L} = \mathcal{L}_C + \mathcal{L}_{V+H} + \mathcal{L}_{N=1}$, where

$$\mathcal{L}_C = -2\int d^4\theta\, V_E(\bar{\Xi}\Xi + \bar{\Xi}^c\Xi^c) - \left\{\int d^2\theta\, (\Xi^c\partial_y\Xi - \Xi\partial_y\Xi^c) + \text{h.c.}\right\}, \quad (18)$$

$$\mathcal{L}_{V+H} = \left\{\int d^2\theta\, \frac{1}{4}E\mathcal{W}\mathcal{W} + \text{h.c.}\right\} + \int d^4\theta\, \frac{E + \bar{E}}{2V_E^2}\left(-\partial_y V - i\Phi_S + i\bar{\Phi}_S\right)^2$$

$$+ \int d^4\theta\, V_E\left(\bar{H}e^{2gV}H + \bar{H}^c e^{-2gV}H^c\right)$$

$$+ \left\{\int d^2\theta\, H^c\left(\partial_y + m\,E - 2ig\Phi_S\right)H + \text{h.c.}\right\},$$

and $\mathcal{L}_{N=1}$ is the boundary Lagrangian for which we omit the explicit expressions. The superfields (Ξ, Ξ^c) and (H, H^c) represent the $N = 1$ chiral multiplets which originate from the compensator and physical hypermultiplets respectively, and the superfields (V, Φ_S) are the $N = 1$ vector and chiral multiplets coming from the physical 5D vector multiplet. The remaining superfields V_E and E are a spurion-like vector superfield coming from the fifth component of the 5D Weyl multiplet and a chiral superfield coming from the central charge vector multiplet, respectively. Especially E corresponds to the radion superfield in the '5D off-shell approach' in the terminology of Ref. [14]. Note that $V_E \simeq (E + \bar{E})/2$ for the vanishing VEVs of physical fields. The mass scales are again measured in the unit of $M_5 \equiv 1$. Note that we have omitted the terms involving graviphoton ($I = 0$) multiplet and a part of Chern-Simons terms, that are irrelevant to the following discussions.

---

[3]The Z_2-odd gauge coupling can be realized in the supergravity through the four-form mechanism [18].
[4]Note that $\omega_3 = 0$ due to $TZT = Z$ in Eq. (2).

The relation between the fields in the superconformal multiplets and the above superfields Ξ, Ξ^c, H, H^c, V, Φ_S, V_E and E is found in the appendix of Ref. [14]. The (untwisted) superconformal gauge fixings (1) for the compensator chiral superfields are rewritten as

$$
\begin{aligned}
\xi &= \sqrt{1 + \frac{1}{2}(|h|^2 + |h^c|^2)}, & \xi^c &= 0, \\
\chi_\xi &= \frac{1}{2\xi}(\bar{h}\chi_h + \bar{h}^c\chi_h^c), & \chi_\xi^c &= -\frac{1}{2\xi}(h\chi_h^c + h^c\chi_h),
\end{aligned}
\tag{19}
$$

where each component field is defined as $\Phi = \phi - \theta\chi_\phi - \theta^2\mathcal{F}_\phi$, $\Phi^c = \phi^c - \theta\chi_\phi^c - \theta^2\mathcal{F}_\phi^c$ ($\Phi = \Xi, H$).

Now we consider the twisted $SU(2)_U$-gauge fixing (7), which corresponds to the SS-twisted Poincaré supergravity. In this case, thanks to our simplified interpretation, the only change is the replacement of the compensator chiral superfields Ξ and Ξ^c by

$$
\begin{pmatrix} \Xi^c \\ \Xi \end{pmatrix} \rightarrow e^{-i\vec{\omega}\cdot\vec{\sigma}f(y)} \begin{pmatrix} \Xi^c \\ \Xi \end{pmatrix}.
\tag{20}
$$

This replacement just generates additional terms in the action,

$$
\mathcal{L}_{\text{twist}} = f'(y)\left\{ (\omega_2 + i\omega_1)\int d^2\theta\,\Xi^2 + (\omega_2 - i\omega_1)\int d^2\theta\,(\Xi^c)^2 + \text{h.c.} \right\},
\tag{21}
$$

which originate from the $\partial_y\Xi$ and $\partial_y\Xi^c$ terms in Eq. (18).

We have to go to the on-shell description here, because Ξ, Ξ^c, and E (as well as the graviphoton multiplet omitted here) become *dependent* superfields after the superconformal gauge fixings [14] (see Eq. (19) for Ξ and Ξ^c). However in the global SUSY limit ($M_5 \rightarrow \infty$), these dependent superfields are reduced to spurion-like superfields,

$$
\Xi \simeq 1 - \theta^2\mathcal{F}_\xi, \qquad \Xi^c \simeq -\theta^2\mathcal{F}_\xi^c, \qquad E \simeq 1 - \theta^2\mathcal{F}_E,
\tag{22}
$$

and we can formally keep the superspace structure. From Eq. (22) together with the additional Lagrangian term (21), we find

$$
\begin{aligned}
\mathcal{L}_C &= -2(|\mathcal{F}_\Xi|^2 + |\mathcal{F}_\Xi^c|^2) - \bar{\mathcal{F}}_E\mathcal{F}_\Xi - \bar{\mathcal{F}}_\Xi\mathcal{F}_E + (\partial_y\mathcal{F}_\Xi^c + \text{h.c.}) \\
&\quad + 2f'(y)\left\{ (\omega_2 + i\omega_1)\mathcal{F}_\Xi + \text{h.c.} \right\} \\
&= -2(|\mathcal{F}_\Xi|^2 + |\mathcal{F}_\Xi^c|^2) - \bar{F}_E\mathcal{F}_\Xi - \bar{\mathcal{F}}_\Xi F_E + (\partial_y\mathcal{F}_\Xi^c + \text{h.c.}),
\end{aligned}
$$

where

$$
F_E \equiv \mathcal{F}_E - 2f'(y)(\omega_2 - i\omega_1).
\tag{23}
$$

Then we conclude that the effect of the SS twist is completely absorbed by the constant shift of $\mathcal{F}_E$ as expected. As discussed in the previous works [2] we can easily obtain the soft SUSY breaking terms for the matter fields in $\mathcal{L}_{V+H}$ by substituting $\mathcal{F}_E = F_E + 2f'(y)(\omega_2 - i\omega_1)$ in the F-component of the spurion-like superfield E. Note that all the terms generated by integrating out F_E, $\mathcal{F}_\Xi$ and $\mathcal{F}_\Xi^c$ are higher order in powers of $1/M_5$,

and the leading order effects of the SS SUSY breaking are obtained just by substituting $\mathcal{F}_E = 2f'(y)(\omega_2 - i\omega_1)$ in $\mathcal{L}_{V+H}$.

Finally we should remark that the correspondence between the SS twist and the constant superpotentials discussed in the previous section is manifest in this superspace description, that is, Eq. (21) is nothing but the $N = 1$ invariant constant superpotential. For the case of the singular gauge fixing (12), this is the boundary constant superpotential, as shown in the previous section at the full superconformal gravity level. (See the relation (13).)

5   Conclusions and discussions

By noticing that the twisted $SU(2)_R$ boundary condition in S^1-compactified 5D Poincaré supergravity is equivalent to the twisted $SU(2)_U$ gauge fixing in 5D conformal supergravity, we have reexamined the SS mechanism of SUSY breaking in the latter terminology. In this case, only the compensator hypermultiplet is relevant to the SS breaking, and various properties of the SS mechanism can be easily understood. We realized the 5D Poincaré supergravity with the SS twist starting from the hypermultiplet compensator formalism of 5D conformal supergravity [7, 8, 9, 10]. Thanks to this simplified interpretation, we can explicitly show the Wilson line interpretation of the SS twist [3], the correspondence between the twist and the constant superpotentials [5], and the quantum inconsistency of the twist in the AdS_5 background [6] at the full supergravity level. We have also derived the $N = 1$ superspace description of the SS twist based on our previous works [12, 13, 14].

We remark that the $N = 1$ superspace description of SS twist is possible only in the '5D off-shell approach' in the terminology of Ref. [14], in which the SS twist is translated into the nonvanishing F-term of the radion superfield E. When we use the word "the radion superfield", we have to specify the context in which we work. For example, we have the radion superfield also in the '4D off-shell approach' of Ref. [14], but the nonvanishing F-term of this radion superfield does not realize the SS twist. This is because this approach is possible only when the background preserves $N = 1$ SUSY. For the SUSY breaking case such as the SS-twisted supergravity, we are forced to work in the 5D off-shell approach, in which the radion superfield is a spurion-like superfield.

As shown in Sec. 3.3, if the compensator is charged under some gauge field including the graviphoton (i.e. the R-symmetry is gauged), the SS twist generates a mass for the corresponding gauge field without the Higgs mechanism, except for the case of ABN type gauging [16]. This is the reason why the SS twist results in an inconsistent theory in the AdS_5 background [6]. However, this does not mean that the constant superpotentials cannot be introduced at the fixed points of orbifold in the AdS_5 background, because the correspondence between the SS twist and the constant superpotentials shown in Sec. 3.2 is only valid *without* the R-gaugings. Indeed the R-gauge field does not receive a mass from the constant superpotential. In such a sense, the constant superpotential belongs to a much wider class of deformation parameter than the SS twist in 5D supergravity on S^1/Z_2 orbifold.

Then it may be interesting if we consider the boundary constant superpotentials with the R-symmetry gauged by the Z_2-odd graviphoton through the Z_2-odd gauge coupling

(GP-FLP type gauging [17]), which can not be resembled by SS twist. This possibility is studied in the global SUSY limit in the first paper of Ref. [2]. If the R-gauge coupling is Z_2-even (ABN type gauging), the SS twist is also allowed as shown in Sec. 3.3, and the combination of the twist and the constant superpotentials may result in some nontrivial on-shell supergravities. We expect that this case may be related to the on-shell models with detuned brane tensions [19], even though we have still some gap between the ABN model and the off-shell formulations as mentioned at the end of Sec. 3.2. On the other hand, for the R-symmetry gauged by the Z_2-even physical vector field [20], we can not introduce any constant superpotential because the superpotential must have nonvanishing R-charge at boundaries in this case.

One of the advantages of our derivation of SS-twisted Poincaré supergravity is that, in addition to its simplicity, it is based on the superconformal formulation which can interpolate various frames (such as the so-called Einstein frame or string frame), or even various on-shell supergravities that are physically different from each other. It is well known that the superconformal formulation is indeed powerful in four dimensions. Our method is expected to be useful if we treat the SS-twisted 5D Poincaré supergravity as an effective theory of further higher-dimensional theory such as, e.g. the eleven-dimensional supergravity. For the eleven-dimensional supergravity compactified on S^1/Z_2 times Calabi-Yau three-fold, it is known that the effective 5D supergravity is derived by introducing two compensator hypermultiplets [9] in 5D conformal supergravity. This will be one of the extension of this paper in future works.

Acknowledgement

A   Invariant action in hypermultiplet compensator formalism

In this appendix we review the invariant action derived in Refs. [7, 8, 9, 10] for the relevant multiplets to this paper.

The action for 5D supergravity on S^1/Z_2 orbifold is given by [9]

$$S \;=\; \int d^4x \int dy \,(\mathcal{L}_b + \mathcal{L}_f + \mathcal{L}_{\mathrm{aux}} + \mathcal{L}_{N=1}),$$

where $\mathcal{L}_b$, $\mathcal{L}_f$, $\mathcal{L}_{\mathrm{aux}}$ and $\mathcal{L}_{N=1}$ are the Lagrangians for the bosonic, fermionic, auxiliary fields and the boundary Lagrangian, respectively given by

$$
\begin{aligned}
e^{-1}\mathcal{L}_b \;=\;& -\frac{1}{2}\mathcal{N}R - \frac{1}{4}\mathcal{N}a_{IJ}F^I_{\mu\nu}F^{\mu\nu J} + \frac{1}{2}\mathcal{N}a_{IJ}\nabla^m M^I \nabla_m M^J + \frac{1}{2}\mathcal{N}^{IJ}\mathcal{Y}^{ij}_I \mathcal{Y}_{Jij} \\
&+ \nabla^m \mathcal{A}^{\bar{\alpha}}_{\ i}\nabla_m \mathcal{A}_\alpha^{\ i} + \mathcal{A}^{\bar{\alpha}}_{\ i}(g^2 M^2)_\alpha^{\ \beta}\mathcal{A}_\beta^{\ i} + \mathcal{N}^{-1}(\mathcal{A}^{\bar{\alpha}i}\nabla_a \mathcal{A}_\alpha^{\ j})^2
\end{aligned}
$$

$$+\frac{1}{8}e^{-1}c_{IJK}\epsilon^{\lambda\mu\nu\rho\sigma}W_\lambda^I\left(F_{\mu\nu}^J F_{\rho\sigma}^K+\frac{1}{2}g[W_\mu,W_\nu]^J F_{\rho\sigma}^K+\frac{1}{10}g^2[W_\mu,W_\nu]^J[W_\rho,W_\sigma]^K\right),$$

$$
\begin{aligned}
e^{-1}\mathcal{L}_f &= -2i\mathcal{N}\bar\psi_\mu\gamma^{\mu\nu\rho}\nabla_\nu\psi_\rho+2i\mathcal{N}a_{IJ}\bar\Omega^I\slashed{\nabla}\Omega^J-2i\bar\zeta^{\bar\alpha}(\slashed{\nabla}+gM)\zeta_\alpha\\
&\quad-2i\mathcal{A}_i^{\bar\alpha}\nabla_n\mathcal{A}_{\alpha j}\bar\psi_m^{(i}\gamma^{mnk}\psi_k^{j)}+2ia_{IJ}\mathcal{A}_i^{\bar\alpha}\nabla_a\mathcal{A}_{\alpha j}\bar\Omega_i^I\gamma^a\Omega_j^J-4i\nabla_n\mathcal{A}_i^{\bar\alpha}\bar\psi_m^i\gamma^n\gamma^m\zeta_\alpha\\
&\quad+\mathcal{N}(\bar\psi_m\psi_n)(\bar\psi_k\gamma^{mnkl}\psi_l+\bar\psi^m\psi^n)+\mathcal{N}(a_{IJ}\bar\Omega^{Ii}\gamma_a\Omega^{Jj})^2-ig\mathcal{N}_I[\bar\Omega,\Omega]^I\\
&\quad+\frac{1}{8}(\bar\psi_k\gamma^{mnkl}\psi_l+2\bar\psi^m\psi^n+a_{JK}\bar\Omega^J\gamma^{mn}\Omega^K+\bar\zeta^{\bar\alpha}\gamma^{mn}\zeta_\alpha)^2\\
&\quad+\frac{i}{4}\mathcal{N}_I F_{mn}^I(W)(\bar\psi_k\gamma^{mnkl}\psi_l+2\bar\psi^m\psi^n+a_{JK}\bar\Omega^J\gamma^{mn}\Omega^K+\bar\zeta^{\bar\alpha}\gamma^{mn}\zeta_\alpha)\\
&\quad+i\mathcal{N}a_{IJ}\bar\psi_m(\gamma\cdot F^I(W)-2\slashed{\nabla}M^I)\gamma^m\Omega^J\\
&\quad-2\mathcal{N}a_{IJ}(\bar\Omega^I\gamma^m\gamma^{nk}\psi_m)(\bar\psi_n\gamma_k\Omega^J)+2\mathcal{N}a_{IJ}(\bar\Omega^I\gamma^m\gamma^n\psi_m)(\bar\psi_n\Omega^J)\\
&\quad+\frac{1}{4}i\mathcal{N}_{IJK}\bar\Omega^I\gamma\cdot F^J(W)\Omega^K-\frac{2}{3}(\bar\Omega^I\gamma^{mn}\Omega^J\bar\psi_m\gamma_n\Omega^K+\bar\psi^i\cdot\gamma\Omega^{Ij}\bar\Omega_{(i}^J\Omega_{j)}^K),
\end{aligned}
$$

$$
\begin{aligned}
e^{-1}\mathcal{L}_{\mathrm{aux}} &= D'(\mathcal{A}^2+2\mathcal{N})-8i\bar\chi^{'i}\mathcal{A}_i^{\bar\alpha}\zeta_\alpha-\frac{1}{2}\mathcal{N}_{IJ}(Y^{Iij}-Y_{\mathrm{sol}}^{Iij})(Y_{ij}^J-Y_{ij\,\mathrm{sol}}^J)\\
&\quad+2\mathcal{N}(v-v_{\mathrm{sol}})^{mn}(v-v_{\mathrm{sol}})_{mn}-\mathcal{N}(V_\mu-V_{\mathrm{sol}\,\mu})^{ij}(V^\mu-V_{\mathrm{sol}}^\mu)_{ij}\\
&\quad+(1-W_\mu^{I=0}W^{I=0\,\mu}/(M^{I=0})^2)(\mathcal{F}_i^{\bar\alpha}-\mathcal{F}_{i\,\mathrm{sol}}^{\bar\alpha})(\mathcal{F}_\alpha^{\,i}-\mathcal{F}_{\alpha\,\mathrm{sol}}^{\,i}),
\end{aligned}
\tag{24}
$$

and

$$
\begin{aligned}
\mathcal{L}_{N=1} &= \sum_{l=0,\pi}\Lambda_l\delta(y-y_l)\Big(-\tfrac{3}{2}[\Sigma\bar\Sigma e^{-K^{(l)}(S,\bar S)/3}]_D\\
&\quad+[f_{IJ}^{(l)}(S)W^{I\alpha}W_\alpha^J]_F+[\Sigma^3 W^{(l)}(S)]_F\Big).
\end{aligned}
\tag{25}
$$

Here we have used some expressions defined as

$$
\begin{aligned}
\mathcal{F}_{i\,\mathrm{sol}}^\alpha &= -M^{I=0}(gt_{I=0})^\alpha{}_\beta\mathcal{A}_i^\beta,\\
V_{\mathrm{sol}\,m}^{ij} &= -\frac{1}{2\mathcal{N}}\left(2\mathcal{A}^{\bar\alpha(i}\nabla_m\mathcal{A}_\alpha^{j)}-i\mathcal{N}_{IJ}\bar\Omega^{Ii}\gamma_m\Omega^{Jj}\right),\\
v_{\mathrm{sol}\,mn} &= -\frac{1}{4\mathcal{N}}\left\{\mathcal{N}_I F_{mn}^I(W)-i\left(6\mathcal{N}\bar\psi_m\psi_n+\bar\zeta^{\bar\alpha}\gamma_{mn}\zeta_\alpha-\frac{1}{2}\mathcal{N}_{IJ}\bar\Omega^I\gamma_{mn}\Omega^J\right)\right\},\\
Y_{\mathrm{sol}}^{Iij} &= \mathcal{N}^{IJ}\mathcal{Y}_J^{ij},\qquad \mathcal{F}_{\mathrm{sol}\,i}^\alpha = M^{I=0}(gt_{I=0})^\alpha{}_\beta\mathcal{A}_i^\beta,\\
\mathcal{Y}_I^{ij} &= 2\mathcal{A}_\alpha^{(i}(gt_I)^{\bar\alpha\beta}\mathcal{A}_\beta^{j)}+i\mathcal{N}_{IJK}\bar\Omega^{Ji}\Omega^{Kj},
\end{aligned}
\tag{26}
$$

and D', $\chi^{i'}$ are shifted auxiliary fields from D, χ^i in the Weyl multiplet respectively for which we omit the explicit form (see Ref. [9]). The notation $A^{(i}B^{j)}$ is defined as

$$
A^{(i}B^{j)}=\frac{1}{2}(A^iB^j+A^jB^i).
\tag{27}
$$

The indices $I,J,\ldots=0,1,2,\ldots,n_V$ label the vector multiplets, and $I=0$ component corresponds to the central charge vector (almost graviphoton) multiplet. The gauge kinetic mixing a_{IJ} is calculated as

$$
a_{IJ}\equiv-\frac{1}{2}\frac{\partial^2}{\partial M^I\partial M^J}\ln\mathcal{N}=-\frac{1}{2\mathcal{N}}\left(\mathcal{N}_{IJ}-\frac{\mathcal{N}_I\mathcal{N}_J}{\mathcal{N}}\right),
$$

Weyl multiplet	
$\Pi = +1$	$e_{\underline{\mu}}{}^{m}$, $e_y{}^4$, $\psi_{\underline{\mu}+}$, ψ_{y-}, ε_+, η_-, $b_{\underline{\mu}}$, $V_{\underline{\mu}}^{(3)}$, $V_y^{(1,2)}$, $v^{4\underline{m}}$, χ_+, D
$\Pi = -1$	$e_{\underline{\mu}}{}^4$, $e_y{}^{\underline{m}}$, $\psi_{\underline{\mu}-}$, ψ_{y+}, ε_-, η_+, b_y, $V_y^{(3)}$, $V_{\underline{\mu}}^{(1,2)}$, $v^{\underline{mn}}$, χ_-
Vector multiplet	
$\Pi(M)$	M, W_y, $Y^{(1,2)}$, Ω_-
$-\Pi(M)$	$W_{\underline{\mu}}$, $Y^{(3)}$, Ω_+
Hypermultiplet	
$\Pi(\mathcal{A}_{i=1}^{2\hat{\alpha}-1})$	$\mathcal{A}_{i=1}^{2\hat{\alpha}-1}$, $\mathcal{A}_{i=2}^{2\hat{\alpha}}$, $\mathcal{F}_{i=2}^{2\hat{\alpha}-1}$, $\mathcal{F}_{i=1}^{2\hat{\alpha}}$, $\zeta_+^{\hat{\alpha}}$
$-\Pi(\mathcal{A}_{i=1}^{2\hat{\alpha}-1})$	$\mathcal{A}_{i=2}^{2\hat{\alpha}-1}$, $\mathcal{A}_{i=1}^{2\hat{\alpha}}$, $\mathcal{F}_{i=1}^{2\hat{\alpha}-1}$, $\mathcal{F}_{i=2}^{2\hat{\alpha}}$, $\zeta_-^{\hat{\alpha}}$

Table 1: The Z_2 parity assignment. The underbar means that the index is 4D one. The subscript $\pm$ of the $SU(2)$ Majorana spinors is defined as, e.g. $\psi_+ = \psi_R^{i=1} + \psi_L^{i=2}$ and $\psi_- = i(\psi_L^{i=1} + \psi_R^{i=2})$ where $\psi_{R,L} = (1 \pm \gamma_5)\psi/2$, except for $\zeta_+^{\hat{\alpha}} = i(\psi_L^{\alpha=2\hat{\alpha}-1} + \psi_R^{\alpha=2\hat{\alpha}})$ and $\zeta_-^{\hat{\alpha}} = \psi_R^{\alpha=2\hat{\alpha}-1} + \psi_L^{\alpha=2\hat{\alpha}}$ ($\hat{\alpha} = 1, \ldots, p+q$).

where $\mathcal{N}$ is the norm function of 5D supergravity explained in Sec. 2. The $n_V + 1$ gauge scalar fields M^I are constrained by $\boldsymbol{D}$ gauge fixing shown in Eq. (1) resulting n_V independent degrees of freedom.

For hyperscalars $\mathcal{A}_i^\alpha$, the index $i = 1, 2$ is the $SU(2)_{\boldsymbol{U}}$-doublet index and $\alpha = 1, 2, \ldots, 2(p+q)$ where p and q are the numbers of the quaternionic compensator and physical hypermultiplets respectively. The notation $\bar{\alpha}$ in the action stands for

$$\mathcal{A}_i^{\bar{\alpha}} = d_\beta{}^\alpha \mathcal{A}_i^\beta, \tag{28}$$

where $d_\alpha{}^\beta \equiv \mathrm{diag}(\mathbf{1}_{2p}, -\mathbf{1}_{2q})$.

It was shown in Ref. [9, 10] that the above off-shell supergravity can be consistently compactified on an orbifold S^1/Z_2 by the Z_2-parity assignment shown in Table 1 without a loss of generality. When the fifth dimension is compactified on such orbifold S^1/Z_2, we generically have the boundary $N = 1$ invariant action $\mathcal{L}_{N=1}$ with the constant $\Lambda_{0,\pi}$ at the orbifold fixed points $(y_0, y_\pi) = (0, \pi R)$ as shown in Eq. (25). In the boundary action, Σ is the 4D $N = 1$ compensator chiral multiplet with the Weyl and chiral weight $(w, n) = (1, 1)$ induced by the 5D compensator hypermultiplet, while S and $W^{I\alpha}$ stand for generic chiral matter and gauge (field strength) multiplets with $(w, n) = (0, 0)$ at the boundaries which come from either bulk fields or pure boundary fields. Here the symbols $[\cdots]_D$ and $[\cdots]_F$ represent the D- and F-term invariant formulae, respectively, in the $N = 1$ superconformal tensor calculus [9].

Without the boundary $N = 1$ action $\mathcal{L}_{N=1}$, the auxiliary fields on-shell are given by $V_\mu = V_{\mathrm{sol}\,\mu}$, $v_{mn} = v_{\mathrm{sol}\,mn}$, $Y^{Iij} = Y_{\mathrm{sol}}^{Iij}$, $\mathcal{F}_i^\alpha = \mathcal{F}_{\mathrm{sol}\,i}^\alpha$, and $\mathcal{L}_{\mathrm{aux}}$ finally vanishes on-shell, $e^{-1}\mathcal{L}_{\mathrm{aux}}^{\mathrm{on\text{-}shell}} = 0$.

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